1 Introduction

In this sixth lecture, two-dimensional pressure driven flow (Poiseuille flow) between two plates and flows in porous media were introduced at the beginning of the lecture. Then, we started Chapter 4 which is passive scalar transport. In this section, we discussed about the transport equation and we talked about diffusion-advection of a species and also their competition in the fluid. This material is covered in Chapter 4 of Kirby. (Ref. [1]).

2 Two dimensional Poiseuille flow between two plates

In our second lecture, we discussed one-dimensional Poiseuille flow which has a parabolic velocity profile and velocity profile of the fluid depends only on y direction. The one-dimensional Poiseuille flow equation between parallel plates from the second lecture is:

$$U(y) = \frac{1}{2\mu} \frac{dP}{dx} \left(y^2 - \frac{h^2}{4}\right) \bar{e}_x.$$  (1)

Figure 1: Flow between parallel plates.
Similarly, in two-dimensional (2-D) pressure driven flow (Poiseuille flow) between two plates, we have a velocity equation with pressure gradients in x and z directions. The velocity equation for two-dimensional (2D) Poiseuille flow between parallel plates is:

\[ U(y) = \frac{1}{2\mu} \nabla P_{2D} \left( y^2 - \frac{h^2}{4} \right) = \frac{1}{2\mu} \frac{\partial P}{\partial x} \left( y^2 - \frac{h^2}{4} \right) \vec{e}_x + \frac{1}{2\mu} \frac{\partial P}{\partial z} \left( y^2 - \frac{h^2}{4} \right) \vec{e}_z. \] (2)

Our flow velocity is maximum at the center and zero on both top and bottom walls because of no-slip conditions on the walls. The equation of mean flow velocity is:

\[ \langle U \rangle = \text{Mean flow velocity} = \frac{1}{h} \int_{-h/2}^{h/2} U \ dy. \] (3)

Plugging our U velocity from two-dimensional Poiseuille flow (2) to this equation (3), we will get our mean velocity which is:

\[ \langle U \rangle = \frac{1}{h^2} \frac{1}{2\mu} \nabla P_{2D} \int_{-h/2}^{h/2} \left( y^2 - \frac{h^2}{4} \right) \ dy = -\frac{h^2}{12\mu} \nabla P_{2D}. \] (4)

3 Flows in porous media

Darcy’s equation:

Darcy’s law is a phenomenologically derived constitutive equation that describes the flow of a fluid through a porous medium:

\[ \langle U \rangle = -\frac{K}{\mu} \nabla P. \] (5)

In this equation, \( \langle U \rangle \) is the average flux (discharge per unit area, with units of length per time, m/s) through a porous medium, is proportional to \( K \) (permeability of the medium, m\(^2\)) and \( \nabla P \) (pressure gradient vector, Pa/m). This value of flow often referred to as the Darcy flux.

Comparing our two-dimensional Poiseuille flow equation (2) and Darcy’s flux equation (5), it can easily be seen that our permeability factor \( K \) for the flow between 2 plates with fixed gap distance is \( K = h^2/12 \). Permeability factor depends on geometry.

On the other hand, if we remember our potential flow properties, which has no vorticity (\( \omega = 0 \)) and velocity is a gradient of potential (\( U = \nabla \phi \)). It can be seen that, Darcy’s flux